Fuel-Containment Requirements for Gaseous-Fuel Nuclear Rockets

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Fuel-containment requirements for open-cycle gaseous-fuel nuclear rockets are examined for systems in which there is physical contact between propellant and fuel. Such systems necessarily allow some loss of nuclear material. Analysis shows that the total fuel lost during the propulsion period is the prime constraint in determining containment requirements. A parameter, the containment factor, is introduced to provide a measure of fuel-containment efficiency. Application to representative high-thrust (booster) and low-thrust (interplanetary) vehicles is considered. The analysis indicates that containment factors fifty times as large as those presently demonstrated experimentally are required, in order to limit nuclear fuel loss during propulsion to the order of 1000 kg.

Nomenclature

a = vehicle acceleration at start of propulsion period, g
a_e = rocket engine thrust-to-weight ratio
A_C = radiating area of gas mixture in cavities
f = solid-fission fraction; the fraction of nuclear power generated in reactor by temperature-limited (i.e., solid) fuel elements
h = enthalpy
I $\equiv I_C/I_S$, ratio of specific impulse at T_C to that at T_S
I_* = specific-impulse ratio which gives $\pi_{pl_{\max}}$
k = Boltzmann constant
$\mathfrak{M}$ = molecular weight
$\dot{m}$ = mass flow rate
n_{FP} = average concentration ratio of nuclear fuel to propellant in rocket exhaust, $n_{FP} \equiv N_{FR}/N_{PR}$
N = average concentration, molecules (or particles)/cm^3; N_{TR} is total molecular concentration in rocket chamber
p = specific cost, dollars/lb
p_R = pressure in rocket chamber, atm
P_r = power rejected by radiator
s = tank-to-propellant weight ratio
T = temperature, °K; T_C is maximum temperature attained by propellant in reactor gas cavities, and T_G is its effective radiating temperature
V = ratio of rocket velocity increment ("burnout velocity") to exhaust velocity of propellant at maximum temperature of reactor solids [Eq. (8)]
W = weight; W_F and W_P are weights of nuclear fuel and propellant, respectively, expended per propulsion period
x = reactor-cavity void fraction
l_N = length of nuclear reactor with length = diameter
β = thermal radiation parameter for transparent gas, $\beta \equiv \sigma\epsilon_C A_C T_C^4/\dot{m}h_S$; $\beta_S \equiv \beta/\epsilon_C$
γ = radiator power fraction [Eq. (7)], $\gamma \equiv P_r/\dot{m}h_S$
ϵ_C = emissivity of gas mixture in cavities
ζ = fraction of fission energy which appears as penetrating radiation (gamma rays, neutrons, etc.) and is attenuated by solid materials in reactor
η = radiator "specific weight" (Ref. 8)
θ = thrust-to-weight ratio of Rover equivalent nuclear rocket engine (Ref. 6)
ϑ = temperature ratio [Eq. (7)] $\vartheta \equiv (T_G - T_S)/(T_C - T_S)$
λ = propellant-to-vehicle gross weight ratio, $\lambda \equiv W_p/W_0$
μ = $f + \zeta(1 - f)$
π_{pl} = rocket payload fraction, $\pi_{pl} \equiv W_{pl}/W_0$
ρ = density
σ = Stefan-Boltzmann constant
τ = dimensionless burning time [Eq. (8)]
ψ = containment factor, $\psi \equiv (\rho_{FC}/\rho_{PC})/(\rho_{FR}/\rho_{PR})$

Subscripts

C = condition in nuclear reactor gas cavities
F = nuclear fuel
G = radiating gas mixture
max = maximum
N = nuclear reactor
0 = gross vehicle
P = propellant
pl = payload
R = condition in rocket chamber
S = solid region of reactor (wall)
T = propellant tank

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I. Introduction

SEVERAL open-cycle gaseous-fuel nuclear reactors of the type required for rocket propulsion have been proposed which involve the use of either fluid-dynamic or fluid-magnetic phenomena for preferentially trapping the nuclear material in the gaseous cavities of the reactor.[1] Whatever the trapping mechanism, the essential requirement is that the fuel be trapped, or contained, in gas phase while the gaseous propellant either flows through (in a diffusion process) or around the fuel. In systems utilizing diffusion,[2] the fission energy is transferred from the nuclear material to the propellant by atomic interaction, whereas in systems with separate fuel and propellant regions, the energy transfer is accomplished principally by thermal radiation from fuel to propellant.[3, 4] In all of these situations the fuel and propellant are in physical contact, and it is to be expected that some of the nuclear material will be swept out of the cavities by the propellant into the exhaust nozzle and thereby lost to the system.

In addition to the apparent biological hazard arising from this discharge of nuclear material into the outer environment, consideration must be given to the question of the degradation in engine performance due to the presence of a high-molecular-weight species in the exhaust and the question of economics imposed by the high cost of nuclear fuel. Each of these considerations imposes separate constraints on the containment requirements.

The purpose of this paper is to examine the implications of these constraints and show how they are related to the reactor and rocket engine characteristics, and how these, in turn, ultimately influence over-all vehicle parameters. It is not the author's intent either to accomplish a detailed

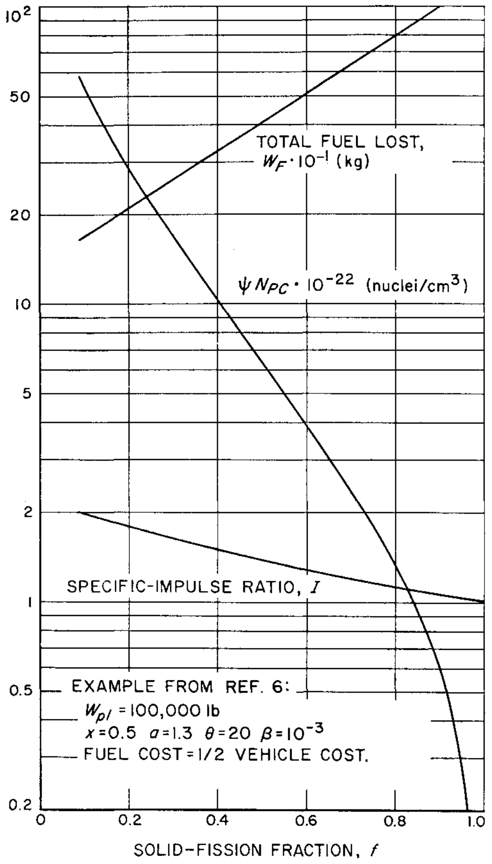

Fig. 1 Fuel-containment requirements for single-stage satellite booster.

systems analysis aimed at comparing various types of propulsion systems or to demonstrate the technical feasiblity and economics of gaseous-fuel nuclear rockets.

II. Containment Factor

For the purposes of this analysis it is convenient to speak in terms of a nuclear fuel containment factor ψ, which is defined as

$$\psi \equiv \frac{\rho_{FC}/\rho_{PC}}{\rho_{FR}/\rho_{PR}} \tag{1}$$

where ρ_{FC} and ρ_{PC} denote, respectively, the average densities of nuclear fuel and propellant in the reactor cavities, and ρ_{FR} and ρ_{PR}, those in the rocket chamber. It is implied that the fuel-propellant mixture leaving the reactor cavities passes into a rocket motor chamber and thence out of the exhaust nozzle. Since the fuel leaving the cavities is lost to the system, a high value of the containment factor ψ is desired.

Another quantity of interest is n_{FP}, the ratio of the average molecular concentration of fuel in the rocket exhaust to that of the propellant:

$$n_{FP} \equiv \frac{N_{FR}}{N_{PR}} = \frac{W_F}{W_P}\frac{\mathfrak{M}_P}{\mathfrak{M}_F} \tag{2}$$

where W_F and W_P are the weights of fuel and propellant expended per propulsion period, and $\mathfrak{M}_F$ and $\mathfrak{M}_P$ are their respective molecular weights. On the basis of rocket-motor performance characteristics alone, it is possible to determine a maximum allowable value for n_{FP}. This constraint arises because of the great disparity in the molecular weights of nuclear fuels and efficient propellants. For plutonium and molecular hydrogen, $\mathfrak{M}_F/\mathfrak{M}_P \simeq 120$; thus in a nuclear rocket engine of given power, the addition of one part by volume of plutonium to 120 parts of pure hydrogen would double the average molecular weight of the exhaust mixture and reduce the specific impulse by about 20%. Therefore, to avoid significant degradation in specific impulse, it is necessary that n_{FP} be $\leq$ 0.001 for plutonium-hydrogen mixtures. However, this upper limit of 0.001 is no constraint at all because it results in the loss of a total mass of nuclear fuel per propulsion period which is 12% of the mass of propellant ejected, a very large amount indeed. From a practical point of view then, containment requirements will be determined not from performance considerations but by the total amount of nuclear fuel one is prepared to expend.

To establish the relationship between the containment factor ψ and the total fuel expended W_F, a relationship is first required between ψ and n_{FP}. This is easily established by writing the quantity ψ in terms of the average concentrations:

$$\psi = N_{FC}/N_{PC}\, n_{FP} \tag{3}$$

where N_{FC} and N_{PC} denote the average concentrations of fuel and propellant, respectively, in the cavities. Thus, all that is required now is the fuel-to-propellant weight ratio in Eq. (2). This is easily established, given a suitable criterion. Since it already has been observed that the total mass of fuel expended is the critical consideration, it is convenient to introduce this criterion in terms of the cost of the fuel. For simplicity, this is expressed in terms of relative cost of the vehicle. That is to say, the criterion for allowable fuel loss is selected on the basis of the fraction of total vehicle cost to be invested in nuclear fuel. Thus, if the criterion selected is the requirement that the fuel expenditure equal the value of the rest of the vehicle (less payload), then the fuel-to-propellant weight ratio may be expressed as

$$\frac{W_F}{W_P} = \frac{p_P}{p_F}\left[1 + s\frac{P_T}{p_P} + \frac{p_N}{p_P}\left(\frac{a}{\lambda a_e}\right)\right] \tag{4}$$

where p is cost per pound, λ is the propellant-to-vehicle gross weight ratio, a_e is the rocket engine thrust-to-weight ratio, a is the vehicle acceleration at the start of the propulsion period, and s is the tank-to-propellant weight ratio; the subscript T denotes tank and N reactor. In the examples that follow, the following values are used: p_F/p_P = 40,000,† p_N/p_P = 800, p_T/p_P = 400, p_F = \$10,000/lb, and s = 0.05, which leads to $W_F/W_P = (0.525 + 20a/\lambda a_e) \times 10^{-3}$.

An indication of typical values of the containment factor and of the corresponding nuclear fuel lost per propulsion period is obtained from two examples in the following sections: 1) a high-thrust, single-stage booster to place a 100,000-lb satellite in Earth orbit and 2) a low-thrust interplanetary vehicle for transferring a 400,000-lb payload from Earth-satellite orbit to a circular orbit about Mars (at 1.1 Mars radius). These two cases illustrate the two basically different propulsion applications possible with gaseous-fuel reactors.

The comparison of containment requirements on the basis of a common mission objective is a convenient measure of the relative investment in nuclear material necessary to achieve improved vehicle characteristics (i.e., increased payload fraction). Previous analyses have shown that the perform-

† The reader may note that these numbers lead to p_P (hydrogen) = \$0.25/lb. Some may argue that, for the low-thrust application (orbit to orbit), the propellant should be valued at the cost to raise it to orbit, i.e., of the order of \$100/lb. If such a figure were used, the resulting p_F/p_P of 400 would lead to larger allowable nuclear fuel losses (lower required containment factors). Therefore, although the figures used herein are arbitrary, the results obtained for the low-thrust application are, by this reasoning, conservative. Other factors beyond the scope of the present study (e.g., vehicle reuse and consequent cost amortization) could also affect the cited examples, but this paper is not intended to demonstrate economic feasibility.

ance of a gaseous reactor as a rocket engine could be controlled by varying the distribution of fissionable material between a conventional mode (e.g., solid-fuel plates) and the gas cavities.[1] This distribution was specified in terms of the solid-fission fraction f, the fraction of the total reactor fission power released in the solid phase. It is convenient to introduce this parameter as the independent variable in the present analysis. A major objective, of course, is to diminish the solid-fission fraction to zero in order to achieve the highest specific-impulse ratio possible. This requires, however, relatively large concentrations of nuclear material in gas phase and therefore large containment factors. Thus, in the final analysis, the efficiency of the containment process determines the maximum fraction of the total fuel mass that can be retained in gas phase and, consequently, vehicle performance. Now, it may very well be that containment factors achievable in practice will be much too small to allow substantial increases in specific impulse over that possible with the "equivalent Rover" reactor.[5] In that event, gas-phase fuel containment will, of course, be uninteresting. The examples that follow indicate that the choice between a high- and a low-thrust application has some bearing on this question.

High-Thrust Application

The satellite mission reported in Ref. 6 is selected for this example. The characteristics of two different single-stage vehicles to perform this mission are summarized in Figs. 9 and 10 of that paper. In the present calculation, we consider only the case in which the thermal radiation parameter $\beta = 10^{-3}$, which represents a gaseous reactor in which the gaseous mixture of fuel-propellant is quite transparent, having emissivities in the order of 10^{-3}–10^{-2}. From the engine and vehicle characteristics given in Ref. 6, the containment factor and fuel lost during propulsion can be directly computed by means of Eqs. (3) and (4). The results are plotted in Fig. 1 as a function of the solid-fission fraction. The corresponding value of the specific-impulse ratio ($I \equiv I_C/I_S$, where I_C and I_S denote the specific impulses corresponding to temperature T_C and T_S) is also shown for convenient reference. The principal trend to note is that, as the amount of fuel in gas phase is increased (i.e., as f is decreased) so as to increase the specific impulse, the containment requirement becomes more severe. This behavior reflects two effects: the higher concentration of nuclear material in gas phase and the smaller total mass (and therefore cost) of the vehicle. Clearly, as more of the fuel is retained in solid form, the containment requirement is rapidly reduced; unfortunately, so also is the performance.

The choice of the quantity ψN_{PC} for summarizing the results in Fig. 1 stems from the fact that the average propellant concentration (i.e., cavity pressure level) is somewhat arbitrary and determined by other considerations. In order to obtain explicit values for ψ, a value for the average propellant density in the cavities N_{PC} must be selected. Table 1 summarizes the results for the present example on the premise that N_{PC} is three times the total particle concentration

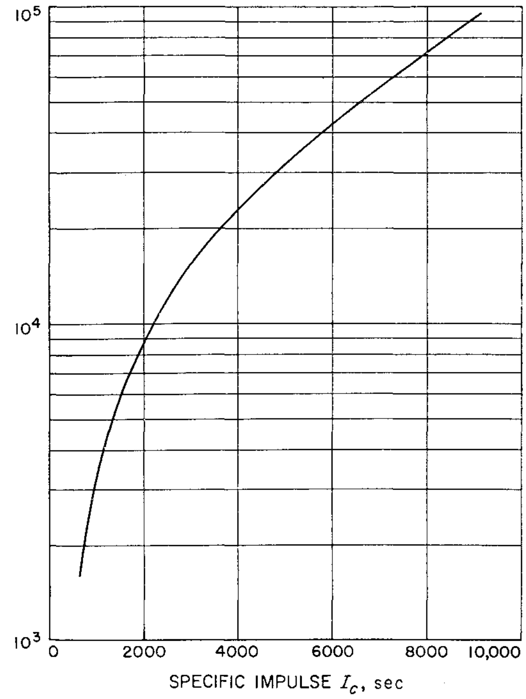

Fig. 2 Temperature-specific impulse relationship for hydrogen.

N_{TR} in the rocket chamber, with

$$N_{TR} = p_R/kT_C \tag{5}$$

The symbol p_R denotes the pressure in the rocket chamber (here taken as 100 atm), k is the Boltzmann constant, and T_C, the temperature to which the propellant is heated in passing through the reactor cavities, corresponds to the specific impulse I_C achieved by gas-phase heating; the relationship between T_C and I_C (Fig. 2) is taken from the work of Altman,[7] and for $T_C > 20{,}000$°K, the perfect gas relation was used assuming complete dissociation and ionization.

On the basis of the parameters selected, the required values of the containment factor ψ increase from about unity for the low-performance systems (i.e., $f \simeq 1$) to values of over a thousand for systems with considerable gas-phase heating. It is of interest to note that actual containment experiments using the vortex method[2] have yielded to date values of $\psi \simeq 1.1$. The results of Table 1 indicate that, regardless of the choice of the average particle concentration in the rocket chamber, values of the containment factor in the order of at least 10 must be attained to allow as much as a 10% increase in the specific impulse over that possible with the equivalent Rover engine. Even then, the attendant loss in nuclear fuel is in the order of 1000 kg per propulsion period. The last column of Table 1 lists the average density ratio of fuel to propellant in the reactor cavities. (This quantity corresponds to $\bar{w}$ in Ref. 2.)

The variation of the containment factor with vehicle size, which results from the arbitrarily selected criterion that the

Table 1 Single-stage satellite booster ($I = 1.14$)[a]

f	$N_{FC} \times 10^{-16}$ (nuclei/cm³)	$\psi N_{PC} \times 10^{-22}$ (nuclei/cm³)	I_C, sec	T_C, °K	$N_{PC} \times 10^{-20}$ (nuclei/cm³)	ψ	ρ_{FC}/ρ_{PC}
0.090	730	58.3	1400	5400	4.03	1440	2.17
0.165	450	34.7	1300	4700	4.63	750	1.166
0.29	252	18.5	1130	3950	5.51	336	0.549
0.50	97	6.39	950	3100	7.02	91	0.166
0.91	9.0	0.533	730	2050	10.62	5.0	0.0102
0.98	1.7	0.100	700	2000	10.9	0.9	0.0019

[a] Parameters[6] W_{pl} = 100,000 lb; $x = 0.5$; $a = 0.3$; $\theta = 20$; $\beta = 10^{-3}$; fuel cost = $\frac{1}{2}$ total vehicle cost; and p_R = 100 atm.

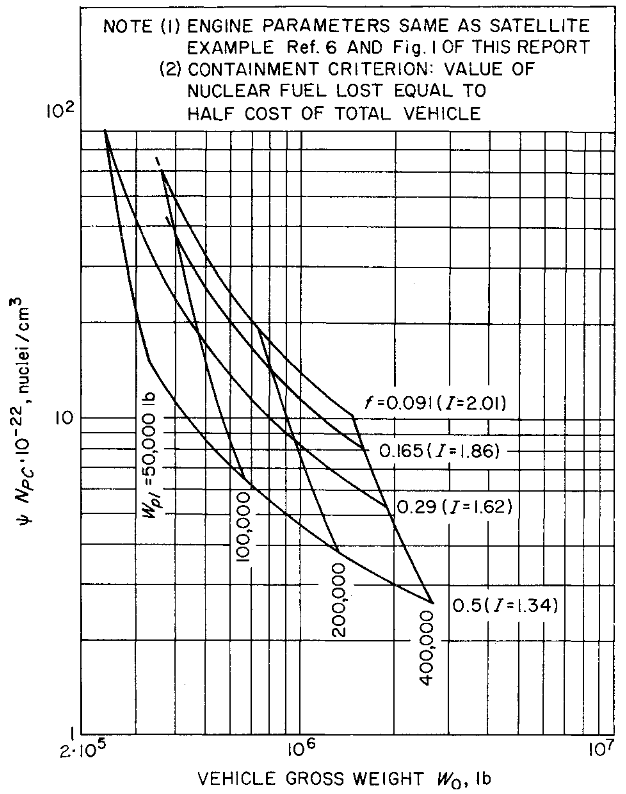

Fig. 3 Influence of vehicle size on containment factor for single-stage satellite booster (V = 1.14).

value of W_F be given by Eq. (4), is shown in Fig. 3 for various values of the solid-fission fraction. The corresponding values of the payload to be placed in Earth orbit are shown in the cross plots. As expected, larger vehicle sizes allow poorer containment. It is noted, however, that even at f = 0.5 (which yields I = 1.34) and a 400,000-lb payload, ψN_{PC} is smaller by only a factor of 2 than in the corresponding case for a 100,000-lb payload. This results in a 2.65-million-lb vehicle and a loss of about 2000 kg of nuclear material per propulsion period. The dependence of ψN_{PC} on vehicle size arises only through the average fuel concentration N_{FC} and reflects simply the variation in reactor size with system size. As the vehicle size increases, at fixed values of the solid-fission fraction, the reactor size increases and the critical fuel concentration decreases.

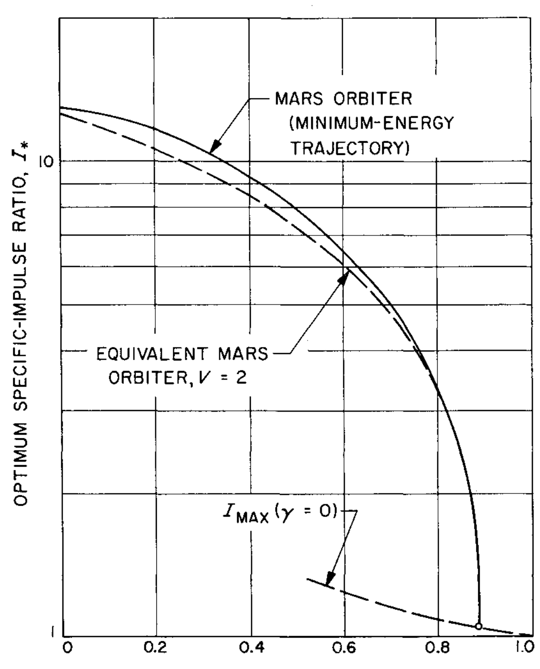

Fig. 4 Optimum specific-impulse ratio as function of solid-fission fraction for Mars orbiter.

Low-Thrust Application

The low-thrust application of gaseous-fuel reactors appears to be most promising for missions to the near planets.[5] On this basis a Mars-orbiter mission was selected as an interesting representative case. The calculations were carried out on two separate bases. In the first, a minimum-energy trajectory was used for the Earth-Mars transfer with planet-centered spiral escape and capture trajectories at each end. In the second, an "equivalent Mars orbiter" analysis was performed using the burnout velocity equation for a single-stage vehicle in gravity- and drag-free flight. This second calculation was performed to allow direct analytical techniques to be applied in relating and examining vehicle performance and reactor critically in terms of vehicle characteristics. In both approaches, however, a single set of engine parameters and a single initial acceleration at Earth orbit ($10^{-3}g$) were selected. Again, as in the high-thrust problem, the solid-fission fraction was taken as the independent variable. In the present calculation, however, the specific impulse was optimized for each value of the solid fission fraction. This was possible because in the low-thrust application a radiator is incorporated into the rocket engine complex.[8] The introduction of a radiator effectively decouples the temperature of the gas mixture in the cavities from the temperature of the engine solids, thereby allowing unlimited increases in specific impulse (at least in principle). Thus the low-thrust application allows an additional degree of freedom in the choice of engine characteristics. The implications in regard to the containment factor in these systems are considered in the discussion that follows.

The engine parameters selected for the minimum-energy Mars trajectory were based on the case used in Ref. 5. These are: θ = 0.3, x = 0.3, σ_a^{FC} = 1000 barns, η = 16, ζ = 0.1, and BeO moderator. In Ref. 5, it was assumed that the gas in the reactor cavities was transparent and that there was a linear relationship between temperature and enthalpy. More recent analyses[9] indicate, however, that for hydrogen propellant, especially in the lower-performance regime, a more realistic relationship is that the temperature varies as the $\frac{2}{3}$ power of the enthalpy. Also, there is a good possibility that a practical fuel-propellant gas mixture might be entirely

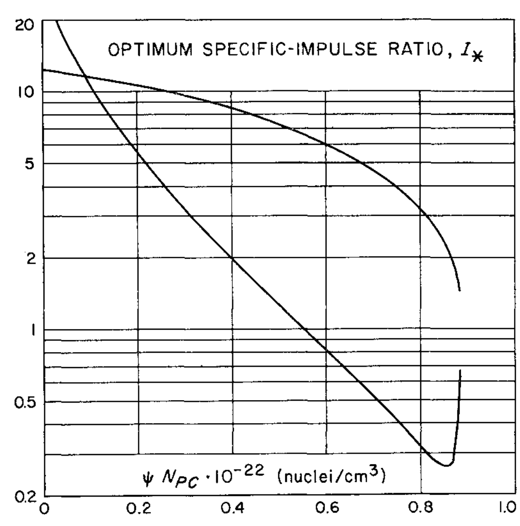

Fig. 5 Fuel-containment requirements for equivalent Mars orbiter (V = 2).

opaque. On this basis, the $\frac{2}{3}$ power law was selected; it was further assumed that the gas mixture in the cavities was opaque to thermal radiation. In general, for a gaseous-reactor system with radiator and an opaque gas in the cavities, the specific-impulse ratio is given by[9]

$$I^2 = 1/\mu[1 + \gamma(1 - \mu) - \beta_S\{[\vartheta(I^{4/3} - 1) + 1^4] - 1\}] \quad (6)$$

where $\mu \equiv f + \zeta(1 - f)$, ζ is the fraction of the fission energy released in the gas phase and not attenuated by the gas mixture, β_S is the thermal radiation parameter divided by the emissivity of the gas in the cavities, ϑ is a temperature ratio, and γ is the radiator power fraction. (See Nomenclature for further details.) Typical solutions for Eq. (6) are shown in Fig. 5 of Ref. 9. The engine performance in terms of the specific-impulse ratio is related to the engine weight by introducing certain dimensionless parameters. The appropriate relations have been derived in Ref. 8, and these are applied directly to the present problem. The principal engine components comprising the gaseous nuclear rocket are the reactor, the radiator, the propellant and tanks, and the payload. All other components are ignored in the weight analysis of the over-all vehicle.

The optimization of the specific-impulse ratio for a given value of the solid-fission fraction is based on the maximization of the payload fraction π_{pl}, given the initial acceleration a and the set of engine parameters in Eq. (6). From Ref. 8,

$$\pi_{pl}(I,\mu) = 1 - (a/\theta I)[(1 + s)\tau(I) + (1 + \eta\theta)\gamma(I,\mu) + 1] \quad (7)$$

where $\gamma(I,\mu)$ is obtained from Eq. (6), and $\tau(I)$, the dimensionless "burning time," is given by

$$\tau(I) = (\theta I/a)(1 - e^{-V/I}) \quad (8)$$

It may be shown that, for this system, the payload fraction passes through a maximum $\pi_{pl_{\max}}$ at $I = I_*$, which is given by the solution to the following equation:

$$1 + (1 + s)\frac{V\theta}{a}e^{-V/I_*} - \frac{1 + \eta\theta}{1 - \mu}\left\{1 + \beta_S + \mu I_*^2 + \beta_S\left(\vartheta + \frac{13}{3}\vartheta I_*^{4/3} - 1\right)[\vartheta(I_*^{4/3} - 1) + 1]^3\right\} = 0 \quad (9)$$

Here $V \equiv \Delta v/c_S$, where Δv is the equivalent velocity increment corresponding to minimum-energy transfer ellipse from Earth to Mars using planet-centered spiral escape and capture trajectories at each end, and c_S is the exhaust velocity corresponding to the maximum solid temperature T_S. For the present example, using $T_S = 2000°K$ and hydrogen propellant, $V = 2$ gives a good approximation for the equivalent Mars orbiter. The solution for I_* from Eq. (9) is shown as the broken-line curve in Fig. 4. The solid-line curve is the actual I_* computed for the minimum-energy transfer ellipse. The agreement is seen to be good.

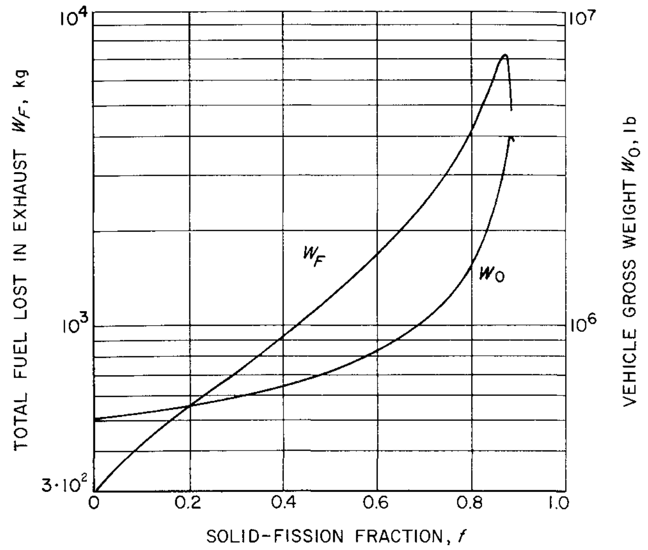

Fig. 6 Fuel lost and vehicle gross weight for equivalent Mars orbiter ($V = 2$).

The results for the optimum specific impulse I_* for a given value of the solid-fission fraction may be used in the set of engine and vehicle parameter relations to determine the characteristics of the optimized system. These results are summarized in Table 2 for the case $V = 2$, using the same choice of the average propellant concentration in the cavities as previously. The containment factor and the optimum specific-impulse ratio are plotted in Fig. 5 as a function of the solid-fission fraction, and Fig. 6 shows the vehicle gross weight and total fuel lost.

The most important trend in these results is the increase in the fuel lost as the containment becomes poorer (i.e., as ψ decreases). For example, at the low value of $\psi N_{PC} = 2.6 \times 10^{22}$, the corresponding fuel lost during a single propulsion period is about 6000 kg. In order to reduce the fuel lost by an order of magnitude, the figures indicate that the containment factor must be increased by a factor of roughly 20. Note that even this case, at the present cost of nuclear material, results in a cost per flight of some 12 million dollars for the fuel alone. Along with the increase in fuel lost with decreasing fractions of gas-phase heating in the reactor, there is an increase in vehicle gross weight (see Fig. 6 and Table 2). This trend, which occurs also in the case of the high-thrust systems (such as the satellite booster discussed earlier), reflects directly the degradation in engine specific impulse with increasing values of the solid-fission fraction (see Fig. 5). Thus, as more of the fissionable material is contained in solid (or any temperature-limited) form, the performance is reduced until at $f = 1$, the limiting value for the specific-impulse ratio of the equivalent Rover system is reached ($I = 1$). This trend in specific impulse is mono-

Table 2 Vehicle and engine characteristics for equivalent Mars orbiter ($V = 2$)[a]

f	I_*	γ	$\pi_{pl_{\max}}$	W_0, kip	l_N, ft	W_N, kip	$n_{FP} \times 10^4$	W_F, kg	$\psi N_{PC} \times 10^{-22}$ (nuclei/cm³)	T_C, °K	ψ	ρ_{FC}/ρ_{PC}
0	12.6	40.4	0.784	511	3.8	5.6	0.750	306	30	89,000	12,250	110
0.440	8.04	69.3	0.602	665	5.8	19.4	1.310	1050	1.665	38,300	293	4.61
0.74	4.28	57.9	0.345	1160	8.0	53	1.213	2860	0.431	15,400	30.5	0.445
0.80	3.39	48.3	0.254	1570	9.0	76	1.078	4100	0.317	11,100	16.1	0.208
0.85	2.50	33.6	0.160	2495	10.4	115	0.834	6220	0.263	7,300	8.8	0.088
0.86	2.28	29.0	0.140	2860	10.7	126	0.750	6804	0.265	6,500	7.9	0.071
0.871	2.03	23.1	0.119	3360	10.9	133	0.633	7238	0.286	5,500	7.2	0.055
0.880	1.62	12.9	0.101	3970	10.3	113	0.409	6243	0.432	3,900	7.8	0.038
0.8815	1.44	8.2	0.100	4010	9.4	82	0.293	4784	0 665	3,400	10.5	0.037
0.890	1.05	0	0.103	3880	4.9	12	0.050	895	...	2,000	...	...

[a] Parameters: $\theta = 0.3$, $\eta = 16$, $x = 0.3$, $\zeta = 0.1$, $\vartheta = 0.1$, $\beta_S = 0.1$, $s = 0.05$, $\sigma_a^{(FC)} = 1000$ b, BeO moderator, $W_{pl} = 400{,}000$ lb, $a = 10^{-3}$.

tonic in f; the effect on the gross weight is also monotonic in the case of the high-thrust systems. In the case of the low-thrust systems, however, a new trend is noted at the high values of f. In the present example of the equivalent Mars orbiter, the gross weight passes through a maximum at $f = 0.881$. This behavior stems from the additional degree of freedom allowed in the engine through the radiator power-fraction parameter [see Eq. (6)]. This influence is apparent in Table 2. It was noted in the earlier discussion that the specific-impulse ratio was optimized for given values of the solid-fission fraction. This yields a specific value for the radiator power fraction γ. At small values of the solid-fission fraction, and, therefore, large values of the specific-impulse ratio, the radiator size is large but the reactor size is small. At large values of the solid-fission fraction, the optimum specific-impulse ratio is near unity and the corresponding radiator requirement is small, but the reactor size is large. Thus, as the solid-fission fraction increases from zero, the total engine weight (radiator plus reactor) at first increases. Beyond a certain point, however, the radiator requirement becomes so small that the total engine weight now decreases. The net result is that the engine thrust-to-weight ratio passes through a minimum and the vehicle gross weight through a maximum.

This behavior may be demonstrated analytically. The problem is to determine the extremum of $\pi_{pl_{\max}}(I^*,\mu)$, given the constraint of Eq. (9). The mathematical analysis is straightforward and yields the set of numerical results:

$$I_* = 1.493 \qquad \mu = 0.8927 \qquad \tau = 331.8$$
$$f = 0.8808 \qquad \gamma = 9.516 \qquad \pi_{pl_{\max}} = 0.09957$$

which correspond to the engine parameters listed at the bottom of Table 2.

The appearance of an extremum in $\pi_{pl_{\max}}$ extends through the other interesting system parameters and may be traced easily in the numerical results of Table 2. Figures 5 and 6 show clearly the presence of a minimum in the containment factor at $f = 0.85$ and a maximum in the total fuel lost W_F at $f = 0.87$. These also may be derived analytically, but not easily. Considerable algebraic complication arises here because in determining the average fuel concentration in the reactor, use must be made of the transcendental criticality relation.

Conclusions

Some general observations and conclusions can be drawn from the results of this analysis.

1) The primary constraint in determining containment requirements is the total fuel loss allowed during the propulsion period.

2) Given a mission, the containment requirements can be directly related to the engine parameters and the allowable fuel lost per propulsion period.

3) In the high-thrust application, gaseous-fuel nuclear rockets offer significant improvement over conventional (temperature-limited) direct nuclear systems when the containment factor exceeds about 50.

4) Similar containment requirements apply to the low-thrust systems if the nuclear fuel loss is to be kept within reasonable bounds and/or the system performance is to be comparable to nuclear-electric systems for near-Earth planet missions.[5]

5) Containment factors, appreciably less than 50, yield fairly large specific-impulse ratios for the low-thrust systems and would be attractive if the attendant loss of nuclear material were acceptable.

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